ESAM 395 -- Selected Topics in Applied Mathematics: Asymptotics

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Description: The goal of this course is to give a modern introduction to mathematical methods for solving hard mathematics problems that arise in the sciences. The main focus will be to explain the process of applied mathematics, namely how to take a hard problem, of the type ordinarily encountered in applications, and gain insight into its important features. Applied Mathematics is a no-holds-barred competition, in which one uses all available tools to understand a problem as much as possible. The approach requires a combination of (a) ``real'' mathematics, comprised of theorems and exact results; (b) courage and skill in making legitimate approximations; and (c) intelligent use of computers to both verify and extend the validity of the approximations. Introduction to "sophisticated" uses of computing packages.

Useful Additional Textbooks

G. F. Carrier, M. Krook and C. E. Pearson, Functions of a Complex Variable, McGraw Hill, 1966; reprinted by Hod Books, Ithaca, 1983, and by Society of Industrial and Applied Mathematics (SIAM), 2005, as Classics in Applied Mathematics 49

C. M. Bender and S. A. Orszag, Advanced Mathematical Methods for Scientists and Engineers, McGraw-Hill, 1978.

E. J. Hinch, Perturbation Methods, Cambridge University Press, 1991.

Grading: Homework (40%); Take home Midterm (30%); Final Project (30%)

Schedule

Week 1: Introduction and overview of how to use Jupyter notebooks for python

Week 2: Introduction to Numerical Asymptotics: Dominant balance and polynomial equations. Dimensions and Dimensional Analysis. Random polynomial equations and eigenvalues

Week 3: Introduction to Numerical Asymptotics: First order ordinary Differential Equations.

Week 4: Is it important for Series to Converge? Higher order Differential Equations. Simple Integrals

Week 5: Connection Problems: Why series don't converge (singularities in the complex plane)?

Nonconvergent (asymptotic) series are often extremely useful. Special Functions: Bessel and linear ODEs.

Week 6: Connection Problems: Connections, numerical and asymptotic. Nonlinear boundary value problems.

Week 7: Partial Differential Equations and Boundary Layers

Week 8: Partial Differential Equations, Ctd. Green functions. Integral transforms of linear equations

Week 9: Partial Differential Equations, ctd. Evaluating integral transforms: Colors of the rainbow Saddle Point Approximations

Week 10: Nonlinear Partial Differential Equations. A nonlinear Diffusion Equation. A reaction diffusion Equation