PROBLEM SET 2 APPLIED MATHEMATICS 201

Due: October 7

(1) **An ODE.** Consider the following first order ordinary differential equation

$$\frac{dy}{dx} = c\left(\left(\frac{1}{3x^2+4x+1}\right)\left(\frac{y^7}{1+y^2}\right) + y^3\right),$$

with the initial condition $y(1) = 2$.

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(a) Derive a simple formula for the behavior of the solution at large x when $c = 1$. Compare your solution to a numerical solution to the differential equation and verify that it is correct.

(b) Now consider the equation with $c = -1$. What happens in this case? Present both analytic arguments and numerical arguments.

(2) *A higher order Differential equation*

Consider the following nonlinear ordinary differential equations.

$$\frac{d^3y}{dx^3} = -\sqrt{y} - \left(\frac{dy}{dx}\right)^2 - x^3(d^2y/dx^2) - \frac{1}{5+7e^x},$$

with initial conditions $y(0) = 1, y'(0) = 1, y''(0) = 2$.

(a) Solve this equation numerically.

(b) Make a plot of the absolute magnitude of each of the terms in the equation (e.g. the terms are y''', $\sqrt{y}$, etc.

(c) Now rationalize as much of the behavior of the solution as you can. How many different regimes are there in a dominant balance sense?

(d) Develop approximate solutions for (a) very small x and (b) for x

(3) *An Integral!*

Consider the following integral.

$$I(\epsilon) = \int_0^{70} \frac{dx}{\left(\epsilon + 10x + e^{x/10}\right)^{3/2}}$$

(a) Develop an analytical expression for $I(\epsilon)$, for $\epsilon > 0$. You do not have to calculate corrections to your initial estimates for $I(\epsilon)$.

(b) Test your theory with numerical simulations.