PROBLEM SET 3: APPLIED MATHEMATICS 201

Due: October 18

(1) *A Laplace Integral*
Consider the following integral

$$I(\lambda) = \int_0^{\pi/2} \frac{1}{1+t^2} e^{-\lambda \sin^2(t)}\, dt.$$

(a) Develop analytical expressions for $I(\lambda)$ for both small and large λ .
(b) Test your theory with numerical simulations.

(2) *An Asymptotic Series* Consider the integral

$$I(\lambda) = \int_0^1 \frac{e^{-\lambda t}}{1+t}\, dt.$$

(a) Using Laplace's method, determine the leading order scaling behaviour as $\lambda \to \infty$
(b) Argue that the integral can be accurately approximated as

$$\int_0^\infty \frac{e^{-\lambda t}}{1+t}\, dt$$

by showing the that error made is exponentially smaller in λ .
(c) Determine an asymptotic expansion by substituting a power series expansion for the integrand and integrating term-by-term
(d) Demonstrate that the series has radius of convergence zero, but that the series is asymptotic