1 Conceptual part of the assignment (Optional and fun)

1. Read the papers in Friction folder and try to gain an understanding of the microscopic origin of friction between two dry surfaces, and the effective models that we use to characterize it.

2. Read the papers in the Rubber Elasticity folder and gain an understanding of the different origins of rubber elasticity versus conventional materials

3. What is the difference between inertial and gravitation mass? Read up on the importance of this concept to Einstein's view of gravity.

4. Who does the non-resonant solution to the force SHO not resemble the resonant solution as $\omega_0 \to \omega$?

5. What is the difference between a material that will fracture/crack under stress, as opposed to those that will deform? Think of the difference between what might happen microscopically when you crack a piece of chalk, or glass, versus what happens when you stress a metal hanger?

6. What is the origin of the linear dependence of damping on velocity in viscous systems, versus the quadratic dependence of the kind experienced by airplanes and boats?

7. Read up on Noether's theorem and understand the "reason" for conservation laws, such as conservation of linear and angular momentum, and conservation of energy.

2 Mathematical part of assignment

1. **Finishing up in-class analysis**

   (a) Explicitly solve the equation $d^2x/dt^2 + \omega_0^2 x = 0$. Note, we just guessed the answer in class.

   (b) Show that the guessed solution for $x(t)$ satisfies the SHO with resonant forcing.

(c) Show that $c_3 = \frac{f_0/m}{\omega_0^2 - \omega^2}$ in the non-resonant forcing SHO.

(d) Confirm that the critically damped homogenous solutions satisfies the governing equations.

2. **SHO**

   (a) Simulate the (simplest) SHO analyzed in class in all the various considered limits and convince yourself that you analytical solution agrees with your numerical results.

   Go to http://www.mathworks.com/help/matlab/examples/numerical-integration-of-differential-equations.html for introduction for solving ODEs in matlab numerically.

   (b) Show that the three damped cases that we considered do not lead to divergences, as observed in the undamped scenarios.

3. **Finite time blow-up**
   Imagine that we could reverse drag, so that it acted in the same direction as velocity. For a particle falling under the effects of gravity ($F_g = mg$), with initial velocity zero, and drag proportional to $|\dot{x}|^2$:

   (a) Write down a second order ODE for the particle's position.

   (b) Write down a first order ODE for the particle's velocity.

   (c) Find the critical time t_c at which the particle's position diverges.

   (d) Sketch the particle's position and velocity versus time.

   (e) If we had instead considered drag proportional to $|\dot{x}|$, how would the answer for t_c have differed? Why would that have been a poor model? Note that $\mathbf{F}_{drag} \propto |\dot{x}|$ is usually appropriate at low Reynolds numbers (low speeds) and $\mathbf{F}_{drag} \propto |\dot{x}|^2$ at higher Reynolds numbers (higher speeds).

4. **Parallel springs**
   Suppose that a mass m is attached to two springs in parallel with spring constants k_1 and k_2 and unstretched lengths l_1 and l_2(see Figure 1):

   (a) What is the equilibrium position of the mass?

   (b) Show that the mass executes simple harmonic motion about its equilibrium position.

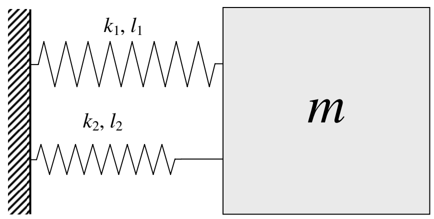

Figure 1: Springs in parallel.

(c) What is the period of oscillation?

(d) If the two springs were to be replaced by one spring, what would be the unstretched length and spring constant of the new spring such that the motion would be equivalent?

5. **Series springs**
   Suppose a mass m is attached to two springs, with spring constants k_1 and k_2, in series (see Figure 2). Answer the same questions as in problem 4. Hint: Apply Newton's law also to the massless point at which the two springs are connected.

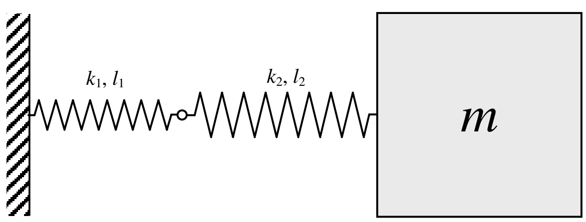

Figure 2: Springs in series.

6. **The probability of heads**

(a) Where in (u, ω) space do real coin tosses lie?

(b) How accurately would you need to control u and ω in the physiologically relevant region to make the outcome of a coin toss entirely predictable?

7. **Chaos**

   (a) Plot the bifurcation diagram of the sine map $f(x) = asin(\pi x)$. $(0 < a < 1)$. What are the similarities with the bifurcation diagram of the logistic map?

   (b) Using the sine map, approximate Feigenbaum?s constants.

   (c) Calculate the Lyapunov exponent as a function of the parameter a for the sine map.?