PROBLEM SET 1: APPLIED MATHEMATICS 201

Due: September 21

(1) *Solving Polynomial Equations, continued* The point of this problem is to consider various modifications and extensions of the polynomial problem $a_1x^5 - a_5x + a_6 = 0$ that we discussed in class (with different labeling of coefficients). Suppose now that the roots x which we seek obey the following equation, $p(x) = a_1x^9 - a_2x^2 + a_3 = 0$.

(a) Nondimensionalize the equation, demonstrating that the solution to the equation depends on a single dimensionless parameter. Call this parameter ϵ . Your nondimensionalized equation should be in the same form as our class example, where ϵ multiplies x^9 .

(b) Closely following the class discussion, construct approximate solutions for all of the roots in the limit as $\epsilon \to 0$ and $\epsilon \to \infty$. You will see that (as in the class example) qualitative transition in the behaviors of the roots between these two limits. You do not have to calculate correction terms for this part.

(c) By hand, compute enough terms in the series expansion to estimate the radius of convergence (do this for the root close to $x = \pm 1$ as $\epsilon \to 0$).

(d) Now numerically calculate the roots and compare the solutions to your formulae. Please present a small number of plots that illustrates that you understand all of the roots.

(e) Finally, *analytically* predict the radius of convergence of the series, to verify your answer. Here you should use the fact that at the radius of convergence two roots collide, so that we simultaneously must satisfy $p(x) = 0$ and $dp/dx = 0$. See the discussion in the class notes.

(2) *Random Polynomials*

Consider the following list of random numbers.

1. (odd) 0.1551 2.833 3.7437 -1.7996 3.3924 2. (even) -1.66 1.80 -4.84 4.60 1.287

You should choose your row (odd/even) depending on the last digit in your birthday. This gives you your fourth order polynomial. For example, if your birthday is on an odd day, you should consider the polynomial in the the first row

$p(x) = 0.1551x^4 + 2.833x^3 + 3.7437x^2 - 1.2996x + 3.3924$

(a) Using a python notebook to compute the roots to your polynomial.

(b) How many dimensionless parameters does a general quartic $p(x) = a_1x^4 + a_2x^3 + a_3x^2 + a_4x + a_5$ have? Nondimensionalize this general equation and then identify the values of these dimensionless parameters for your problem. (Hint: use the Buckingham Pi theorem)

(c) Now explain as much as you can. Develop approximate formulae for all of the roots and see how accurate you can become!

(3) *Dimensional Analysis!*

(a) Upon looking at a table of electrical conductivities of common metals, it is apparent that all metals have the same value, about $1x10^7$ S/m (or $0.1x10^6$ S/cm). See for example this reference
This being notable, please figure out from dimensional analysis where this comes from. Does your dimensional value correspond to the highest conductivity you can image, the lowest or somewhere in between? Please comment.

(b) Invent and solve your own problem in dimensional analysis.