PROBLEM SET 5 APPLIED MATHEMATICS 201

Due: November 18

(1) Write a project plan for your final project. Your project plan should contain a detailed list of "to do" item, each of which is in itself achievable, and the items should be small enough that they can each be sorted out relatively quickly. Besides turning this project plan in wit this homework, you should also add it to your previous document that you shared with Michael, so you can have a conversation about what you are going to do.

(2) Let's consider a random walker with probability $p = 1/7$ of moving to the right and $q = 1/5$ of moving to the left. Since $p + q \neq 1$ there is some probability that the walker stands still.
   (a) Derive the partial differential equation that describes the cloud of walkers. What is the diffusion constant and what is the advection speed?
   (b) Simulate the cloud of walkers. Compare the results of the simulation *quantitatively* to the solution to the diffusion equation we described in class.

(3) Consider the integral

$$I(x) = \int_0^1 dt \ \exp(ix(t - t^3))$$

   (a) Evaluate the integrals numerically as a function of x , making sure to push the software to the limit where it fails. Make a log-log plot of $I(x)$ versus x , focusing on the limit where x is large.
   (b) Now derive a theory for the observed behavior using the method of stationary phase, and compare your theory to the observed behavior.
   (c) *Bonus problem*: Improve upon your stationary phase solution using the steepest descent method. Namely:
      (i) Find contours in the complex plane upon which the phase $ix(t-t^3)$ are constant. Show the contour integral that results from using these contours.
      (ii) Evaluate the integrals along these contours in the limit where $x \to \infty$ and compare your answer to the numerical solutions, and to your answer from the method of stationary phase.

(4) Consider the nonlinear partial differential equation

$$\partial_t u = \partial_{xx} u + \frac{u^3}{1+u},$$

with the initial conditions

$$u(x,0) = Ae^{-x^2},$$

where $A > 0$. We are interested in the solution for $-\infty \leq x \leq \infty$, with the boundary conditions that $u \to 0$ as $x \to \pm\infty$.

(a) Solve the equation numerically. Find the range of A where the solution blows up and where it doesnt blow up.

(b) When the solution blows up, measure the characteristics of the blow up– how does u behave? What is the characteristic width of the solution?

(c) For the case where u blows up, derive an approximate solution for $u(x,t)$, and quantitatively compare with the simulations. Do the same for the case where u does not blow up.