

Miscellaneous Integrals (Answers)

Instructions: For each of the following, you should be able to perform each of the integrations and differentiate the result to get back to the original integrand **without any errors**.

- $\int \frac{1}{(1+x)\sqrt{x}} dx = 2 \tan^{-1}(\sqrt{x}) + C$
- $\int \frac{\sec^2 x}{1 + \tan x} dx = \ln(1 + \tan x) + C$
- $\int \sin x \sec x dx = -\ln(\cos x) + C$
- $\int \frac{\csc x \cot x}{1 + \csc^2 x} dx = -\tan^{-1}(\csc x) + C$
- $\int \frac{\tan x}{\cos^2 x} dx = \frac{1}{2} \sec^2 x + C$
- $\int \csc^4 x dx = -\frac{\cos x}{3 \sin^3 x} - \frac{2 \cos x}{3 \sin x} + C$
- $\int x \tan^2 x dx = -\frac{x^2}{2} + x \tan x - \frac{1}{2} \ln(1 + \tan^2 x) + C$
- $\int x^2 \cos^2 x dx = \frac{x^2}{2} \cos x \sin x + \frac{x^3}{6} + \frac{1}{2} x \cos^2 x - \frac{1}{4} \cos x \sin x - \frac{x}{4} + C$
- $\int x^5 \sqrt{2-x^3} dx = -\frac{2}{45} (2-x^3)(3x^3+4)\sqrt{2-x^3} + C$
- $\int \frac{1}{\sqrt{x^2+4}} dx = \sinh^{-1}\left(\frac{x}{2}\right) + C$
- $\int \frac{x^2}{\sqrt{25+x^2}} dx = \frac{x}{2} \sqrt{25+x^2} - \frac{25}{2} \sinh^{-1}\left(\frac{x}{5}\right) + C$
- $\int (\cos x) \sqrt{4-\sin^2 x} dx = \frac{1}{2} \sin x \sqrt{4-\sin^2 x} + 2 \sin^{-1}\left(\frac{1}{2} \sin x\right) + C$
- $\int \frac{1}{x^2-x+1} dx = \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + C$
- $\int \sqrt{x^2+x+1} dx = \frac{2x+1}{4} \sqrt{x^2+x+1} + \frac{3}{8} \sinh^{-1}\left(\frac{2}{\sqrt{3}}\left(x+\frac{1}{2}\right)\right) + C$
- $\int \frac{5x+31}{3x^2-4x+11} dx = \frac{5}{6} \ln(3x^2-4x+11) + \frac{103}{87} \sqrt{29} \tan^{-1}\left(\frac{\sqrt{29}}{58}(6x-4)\right) + C$
- $\int \frac{x^4+1}{x^2+1} dx = \frac{x^3}{3} - x + 2 \tan^{-1} x + C$
- $\int \sqrt{x^4+x^7} dx = \frac{2\sqrt{x^4+x^7}(1+x^3)}{9x^2} + C$
- $\int \frac{\sqrt{x}}{1+x} dx = 2\sqrt{x} - 2 \tan^{-1}(\sqrt{x}) + C$
- $\int \frac{\cos x}{\sqrt{4-\sin^2 x}} dx = \sin^{-1}\left(\frac{\sin x}{2}\right) + C$

20. $\int \frac{\cos 2x}{\cos x} dx = 2 \sin x - \ln(\sec x + \tan x) + C$
21. $\int \frac{\tan x}{\ln(\cos x)} dx = -\ln(\ln(\cos x)) + C$
22. $\int \frac{x^7}{\sqrt{1-x^4}} dx = -\frac{x^4+2}{6} \sqrt{1-x^4} + C$
23. $\int \ln(1+x) dx = (x+1)(\ln(x+1)-1) + C$
24. $\int x \sec^{-1} x dx = \frac{x^2}{2} \sec^{-1} x - \frac{1}{2} \sqrt{x^2-1} + C$
25. $\int \sqrt{x^2+9} dx = \frac{x}{2} \sqrt{x^2+9} + \frac{9}{2} \sinh^{-1} \left(\frac{x}{3} \right) + C$
26. $\int \frac{x^2}{\sqrt{4-x^2}} dx = -\frac{x}{2} \sqrt{4-x^2} + 2 \sin^{-1} \left(\frac{x}{2} \right) + C$
27. $\int \sqrt{2x-x^2} dx = \frac{1}{2}(x-1)\sqrt{2x-x^2} + \frac{1}{2} \sin^{-1}(x-1) + C$
28. $\int \frac{4x-2}{x^3-x} dx = 2 \ln x + \ln(x-1) - 3 \ln(x+1) + C$
29. $\int \frac{x^4}{x^2-2} dx = \frac{x^3}{3} + 2x - 2\sqrt{2} \tanh^{-1} \left(\frac{x}{\sqrt{2}} \right) + C$
30. $\int \frac{\sec x \tan x}{\sec x + \sec^2 x} dx = \ln(\sec x) + \ln(x-1) - 3 \ln(x+1) + C$
31. $\int \frac{x}{(x^2+2x+2)^2} dx = -\frac{x+2}{2(x^2+2x+2)} - \frac{1}{2} \tan^{-1}(x+1) + C$
32. $\int \frac{x^{1/3}}{x^{1/2}+x^{1/4}} dx = \frac{6}{5}x^{5/6} - \frac{12}{7}x^{7/12} + 3x^{1/3} - 12x^{1/12} + 4 \ln(1+x^{1/12}) - 2 \ln(x^{1/6} - x^{1/12} + 1) + 4\sqrt{3} \tan^{-1} \left(\frac{1}{\sqrt{3}}(2x^{1/12} - 1) \right) + C$
33. $\int \frac{1}{1+\cos 2x} dx = \frac{1}{2} \tan x + C$
34. $\int \frac{\sec x}{\tan x} dx = \ln(\csc x - \cot x) + C$
35. $\int \sec^3 x \tan^3 x dx = \frac{3-5 \cos^2 x}{15 \cos^5 x} + C$
36. $\int x^2 \tan^{-1} x dx = \frac{x^3}{3} \tan^{-1} x - \frac{x^2}{6} + \frac{1}{6} \ln(x^2+1) + C$
37. $\int x(\ln x)^3 dx = \frac{x^2}{2} \ln^3 x - \frac{3}{4}x^2 \ln^2(x) + \frac{3}{4}x^2 \ln x - \frac{3x^2}{8} + C$
38. $\int \frac{1}{x\sqrt{1+x^2}} dx = -\tanh^{-1} \left(\frac{1}{\sqrt{x^2+1}} \right) + C$
39. $\int e^x \sqrt{1+e^{2x}} dx = \frac{e^x}{2} \sqrt{1+e^{2x}} + \frac{1}{2} \sinh^{-1}(e^x) + C$

40. $\int \frac{x}{\sqrt{4x-x^2}} dx = -\sqrt{4x-x^2} + 2\sin^{-1}\left(-1 + \frac{x}{2}\right) + C$
41. $\int \frac{1}{x^3\sqrt{x^2-9}} dx = \frac{1}{18} \frac{\sqrt{x^2-9}}{x^2} - \frac{1}{54} \arctan\left(\frac{3}{\sqrt{x^2-9}}\right) + C$
42. $\int \frac{x}{(7x+1)^{17}} dx = \frac{1}{784} (7x+1)^{-16} - \frac{1}{735} (7x+1)^{-15} + C$
43. $\int \frac{4x^2+x+1}{4x^3+x} dx = \ln x + \frac{1}{2} \tan^{-1}(2x) + C$
44. $\int \frac{4x^3-x+1}{x^3+1} dx = 4x - \frac{2}{3} \ln(x+1) + \frac{1}{2} \ln(x^2-x+1) - \frac{4}{\sqrt{3}} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + C$
45. $\int \tan^2 x \sec x dx = \frac{1}{2} \sin x \tan^2 x + \frac{1}{2} \sin x - \frac{1}{2} \ln(\sec x + \tan x) + C$
46. $\int \frac{x^2+2x+2}{(x+1)^3} dx = \frac{-1}{2(x+1)^2} + \ln(x+1) + C$
47. $\int \frac{x^4+2x+2}{x^5+x^4} dx = -\frac{2}{3x^3} + \ln(x+1) + C$
48. $\int \frac{8x^2-4x+7}{(x^2+1)(4x+1)} dx = 2\ln(4x+1) - \tan^{-1} x + C$
49. $\int \frac{3x^5-x^4+2x^3-12x^2-2x+1}{(x^3-1)^2} dx = \frac{1}{x-1} + \ln(x-1) + \ln(x^2+x+1) - \frac{2}{x^2+x+1} + C$
50. $\int \frac{x}{x^4+4x^2+8} dx = \frac{1}{4} \tan^{-1}\left(\frac{x^2}{2} + 1\right) + C$
51. $\int (\ln x)^6 dx = x(\ln x)^6 - 6x(\ln x)^5 + 30x(\ln x)^4 - 120x(\ln x)^3 + 360x(\ln x)^2 - 720x \ln x + 720x + C$
52. $\int \frac{(1+x^{2/3})^{3/2}}{x^{1/3}} dx = \frac{3}{5} (1+x^{2/3})^{5/2} + C$
53. $\int \frac{(\arcsin x)^2}{\sqrt{1-x^2}} dx = \frac{1}{3} (\sin^{-1} x)^3 + C$
54. $\int \frac{1}{x^{3/2}(1+x^{1/3})} dx = \frac{-2}{\sqrt{x}} + \frac{6}{x^{1/6}} + 6 \tan^{-1}(x^{1/6}) + C$
55. $\int \tan^3 x dx = \frac{1}{2} \tan^2 x - \frac{1}{2} \ln(1 + \tan^2 x) + C$
56. $\int \sin^2 x \cos^4 x dx = -\frac{1}{6} \sin x \cos^5 x + \frac{1}{24} \cos^3 x \sin x + \frac{1}{16} \cos x \sin x + \frac{1}{16} x + C$
57. $\int \frac{xe^{x^2}}{1+e^{2x^2}} dx = \frac{1}{2} \tan^{-1}(e^{x^2}) + C$
58. $\int \frac{\cos^3 x}{\sqrt{\sin x}} dx = -\frac{2}{5} (\sin x)^{5/2} + 2\sqrt{\sin x} + C$
59. $\int x^3 e^{-x^2} dx = -\frac{1}{2} e^{-x^2} (x^2 + 1) + C$
60. $\int \sin \sqrt{x} dx = 2 \sin(\sqrt{x}) - 2\sqrt{x} \cos(\sqrt{x}) + C$

61. $\int \frac{\arcsin x}{x^2} dx = -\frac{\sin^{-1} x}{x} - \tanh^{-1} \left(\frac{1}{\sqrt{1-x^2}} \right) + C$
62. $\int x^2 \sqrt{1-x^2} dx = -\frac{x}{4}(1-x^2)^{3/2} + \frac{x}{8}\sqrt{1-x^2} + \frac{1}{8}\sin^{-1} x + C$
63. $\int \sqrt{x^2-9} dx = \frac{x}{2}\sqrt{x^2-9} - \frac{9}{2}\ln(x+\sqrt{x^2-9}) + C$
64. $\int x\sqrt{2x-x^2} dx = -\frac{1}{2}(2x-x^2)^{3/2} + \frac{x-1}{2}\sqrt{2x-x^2} + \frac{1}{2}\sin^{-1}(x-1) + C$
65. $\int \frac{x-2}{4x^2+4x+1} dx = \frac{5}{4(1+2x)} + \frac{1}{4}\ln(1+2x) + C$
66. $\int \frac{2x^2-5x-1}{x^3-2x^2-x+2} dx = 2\ln(x-1) - \ln(x-2) + \ln(x+1) + C$
67. $\int \frac{e^{2x}}{e^{2x}-1} dx = \frac{1}{2}\ln(e^{2x}-1) + C$
68. $\int \frac{\cos x}{\sin^2 x - 3\sin x + 2} dx = -\ln(-1+\sin x) + \ln(-2+\sin x) + C$
69. $\int \frac{2x^3+3x^2+4}{(x+1)^4} dx = -\frac{5}{3(x+1)^3} + \frac{3}{x+1} + 2\ln(x+1) + C$
70. $\int \frac{\sec^2 x}{\tan^2 x + 2\tan x + 2} dx = \tan^{-1}(1+\tan x) + C$
71. $\int \frac{x^3+x^2+2x+1}{x^4+2x^2+1} dx = -\frac{1}{2(x^2+1)} + \frac{1}{2}\ln(x^2+1) + \tan^{-1} x + C$
72. $\int \sin x \cos 3x dx = -\frac{1}{8}\cos(4x) + \frac{1}{4}\cos(2x) + C$
73. $\int x^5 \sqrt{x^3-1} dx = \frac{2}{45}(3x^6-x^3-2)\sqrt{x^3-1} + C$
74. $\int \ln(x^2+2x) dx = x\ln(x^2+2x) - 2x + 2\ln(x+2) + C$
75. $\int \frac{\sqrt{1+\sin x}}{\sec x} dx = \frac{2}{3}(1+\sin x)^{3/2} + C$
76. $\int \frac{1}{x^{2/3}(1+x^{2/3})} dx = 3\tan^{-1}(x^{1/3}) + C$
77. $\int \frac{\sin x}{\sin 2x} dx = \frac{1}{2}\ln(\sec x + \tan x) + C$
78. $\int \sqrt{1+\cos x} dx = 2\sqrt{2}\sin\left(\frac{x}{2}\right) + C$
79. $\int \sqrt{1+\sin x} dx = \frac{2(-1+\sin x)\sqrt{1+\sin x}}{\cos x} + C$
80. $\int \frac{\sec^2 x}{1-\tan^2 x} dx = -\frac{1}{2}\ln\left(\tan^2\left(\frac{x}{2}\right) + 2\tan\left(\frac{x}{2}\right) - 1\right) + \frac{1}{2}\ln\left(\tan^2\left(\frac{x}{2}\right) - 2\tan\left(\frac{x}{2}\right) - 1\right) + C$
81. $\int \ln(x^2+x+1) dx = x\ln(x^2+x+1) - 2x + \frac{1}{2}\ln(x^2+x+1) + \sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) + C$

82. $\int e^x \sin^{-1}(e^x) dx = e^x \sin^{-1}(e^x) + \sqrt{1 - e^{2x}} + C$
83. $\int \frac{\arctan x}{x^2} dx = -\frac{\tan^{-1} x}{x} + \ln x - \frac{1}{2} \ln(x^2 + 1) + C$
84. $\int \frac{x^2}{\sqrt{x^2 - 25}} dx = \frac{x}{2} \sqrt{x^2 - 25} + \frac{25}{2} \ln(x + \sqrt{x^2 - 25}) + C$
85. $\int \frac{x^3}{(x^2 + 1)^2} dx = \frac{1}{2(x^2 + 1)} + \frac{1}{2} \ln(x^2 + 1) + C$
86. $\int \frac{1}{x\sqrt{6x - x^2}} dx = -\frac{\sqrt{6x - x^2}}{3x} + C$
87. $\int \frac{3x + 2}{(x^2 + 4)^{3/2}} dx = \frac{x - 6}{2\sqrt{x^2 + 4}} + C$
88. $\int x^{3/2} \ln x dx = \frac{2}{5} x^{5/2} \ln x - \frac{4}{25} x^{5/2} + C$
89. $\int \frac{\sqrt{1 + \sin^2 x}}{\sec x \csc x} dx = \frac{1}{3} (1 + \sin^2 x)^{3/2} + C$
90. $\int \frac{e^{\sqrt{\sin x}}}{\sec x \sqrt{\sin x}} dx = 2e^{\sqrt{\sin x}} + C$
91. $\int x e^x \sin x dx = -\frac{1}{2} e^x (x \cos x - \cos x - x \sin x) + C$
92. $\int x^2 e^{x^{3/2}} dx = \frac{2}{3} (x^{3/2} - 1) e^{x^{3/2}} + C$
93. $\int \frac{\arctan x}{(x-1)^3} dx = -\frac{\tan^{-1} x}{2(x-1)^2} - \frac{1}{4(x-1)} - \frac{1}{4} \ln(x-1) + \frac{1}{8} \ln(x^2 + 1) + C$
94. $\int \ln(1 + \sqrt{x}) dx = (x-1) \ln(1 + \sqrt{x}) + \frac{3}{2} + \sqrt{x} - \frac{x}{2} + C$
95. $\int \frac{2x + 3}{\sqrt{3 + 6x - 9x^2}} dx = -\frac{2}{9} \sqrt{3 + 6x - 9x^2} + \frac{11}{9} \sin^{-1} \left(\frac{3x - 1}{2} \right) + C$
96. $\int \frac{1}{\sqrt{e^{2x} - 1}} dx = \tan^{-1} (\sqrt{e^{2x} - 1}) + C$
97. $\int \frac{x^4}{(x-1)^2} dx = \frac{x^3}{3} + x^2 + 3x - \frac{1}{x-1} + 4 \ln(x-1) + C$
98. $\int x^{3/2} \tan^{-1}(\sqrt{x}) dx = \frac{2}{5} x^{5/2} \tan^{-1}(\sqrt{x}) - \frac{x^2}{10} + \frac{x}{5} - \frac{1}{5} \ln(x+1) + C$
99. $\int \sec^{-1}(\sqrt{x}) dx = x \sec^{-1}(\sqrt{x}) - \sqrt{x-1} + C$
100. $\int x \sqrt{\frac{1-x^2}{1+x^2}} dx = \frac{1}{2} \sqrt{1-x^4} + \frac{1}{2} \sin^{-1}(x^2) + C$